A Parallel ½-approx Weighted Matching Algorithm

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CSCAPES Seminar. 16 September, 2008.

Outline

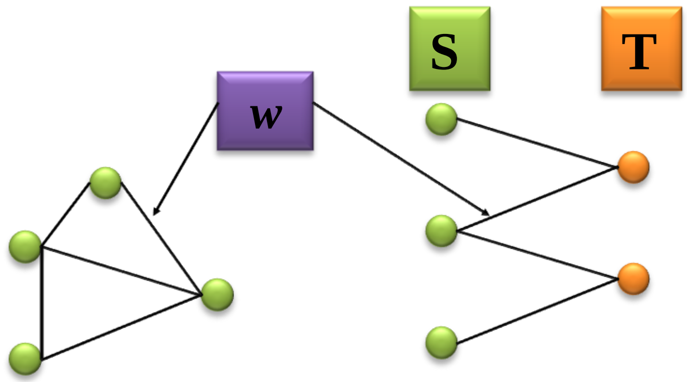
1. Introduction
2. Brief Survey of Parallel Matching Algorithms
3. A ½-approx Parallel Matching Algorithm
4. Computational Results
5. Conclusions and Future work

Graph

A graph G is a pair (V, E)

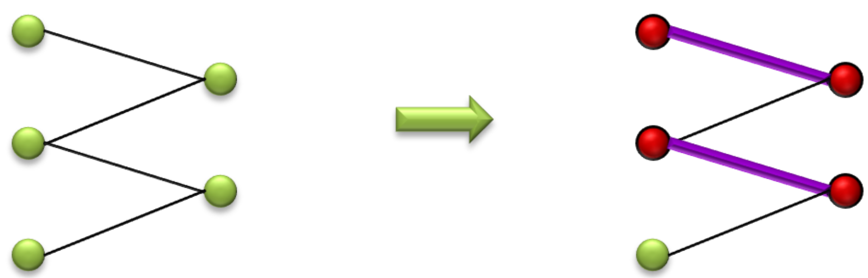
- V is a set of vertices
- E is a set of edges that represent a binary relation on V.

  – Nonbipartite / Bipartite
  – Weighted / Unweighted



Matching

Given a graph, a matching M is a subset of edges such that no two edges in M are incident on the same vertex.

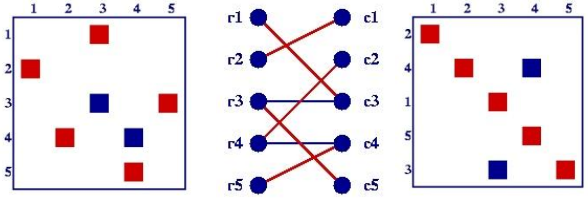


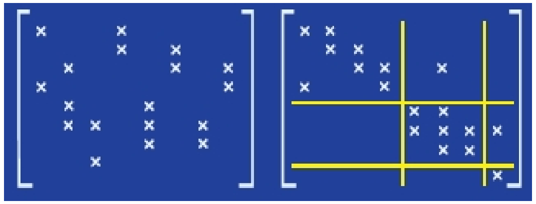
Types:

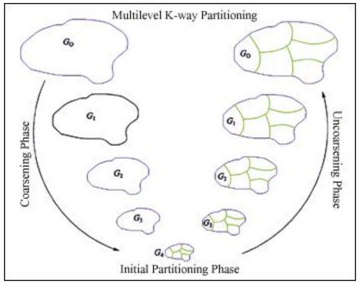
- Maximum Cardinality Matching (no weights)
- Maximum Weight Matching (sum of weights)

Applications of Matchings

- Sparse matrix computations
  - Matrix preconditioning
  - Block Triangular Form
- Multilevel Graph Algorithms
  - Graph partitioners
  - Graph clustering
- Scheduling Problem
  - High speed network switching
  - Facility scheduling problem
- Bioinformatics
  - Homology detection
  - Structural alignment







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A Brief Survey of Parallel Matching Algorithms

- Bipartite Graphs:
  - Auction-based algorithms
  - Augmentation-based algorithms
- Nonbipartite Graphs:
  - Augmentation-based algorithms

Note: Non-exhaustive survey

Auction-based Algorithms

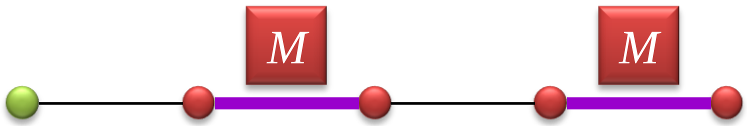

- Primary work:
  - Dimitri P. Bertsekas, MIT
- Basic idea:
  - Buyers bid for objects
  - Iterative process
  - Two basic approaches:
    - Gauss-Seidel: one buyer at a time
    - Jacobi: all buyers bid concurrently
  - Reverse auctions for asymmetric problems
  - Combined forward/reverse (hybrid) approaches for performance

Auction-based Algorithms

- Parallel work:
  - 1979: Bertsekas
  - 1989: Bertsekas and Castanon
  - 1989: Kempka, Kennington and Zaki (Alliant FX/8)
  - 1990: Wein and Zenios : (Connection Machine, CM2)
  - 1992: Goldberg, Plotkin, Shmoys and Tardos (interior point methods)
  - 2003: Reidy and Demmel (In the context of sparse direct solvers – SuperLU)

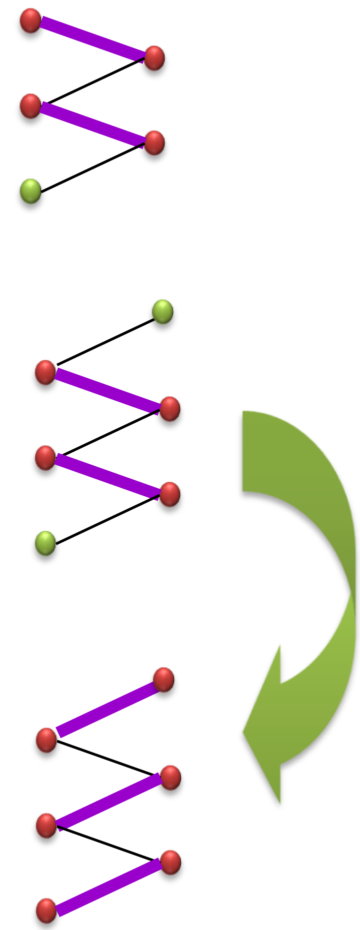
Augmentation

- An alternating path:

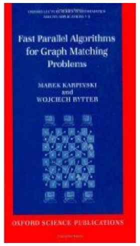


- An augmenting path:

- Augmentation by Symmetric Difference $\oplus$:



Augmentation-based algorithms



- Book: 1998: *Fast Parallel Algorithms for Graph Matching Problems*. Marek Karpinski and Wojciech Rytter. Oxford Science Publications.
- 1993: Goldberg, Plotkin and Vaidya
- 1997: Stor∅y and S∅revik (MasPar MP1 and MP2)
- 1998: Haglin
- 1999: Gupta and Ying (vertex separators)
- 2006: Hougardy and Vinkemeier (path growing, ½-approx)
- 2008: Chan, Dehne, Bose, Latzel (coarse grained algorithms for convex bipartite graphs and trees)

Theoretical in nature.

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   - Implementation Details
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A Serial ½-approx Algorithm: Global

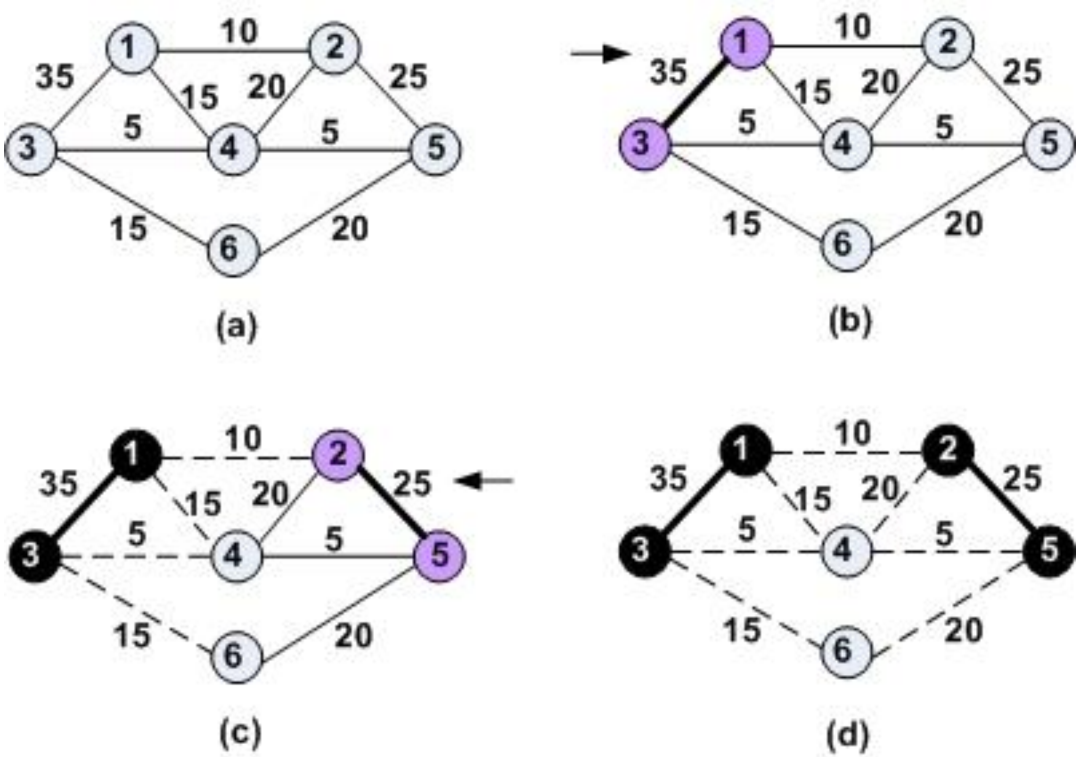
- Sort-based (Avis): $O(|E| \log |E|)$

Algorithm 6 Input: A graph G. **Output:** a matching M. **Effect:** computes a $\frac{1}{2}$-approx matching M in G.

```
procedure GLOBAL-HEAVY(G = (V, E), w : E → R⁺, M)
    M ← φ;
    repeat
        Pick a globally heaviest edge e_uv ∈ E;
        M ← M ∪ e_uv;
        Delete all edges incident on u and v from E;
    until E = φ;
end procedure
```

A Serial ½-approx Algorithm: Global

- Sample execution of sort-based algorithm:



Sequential in nature.

A Serial ½-approx Algorithm: Local

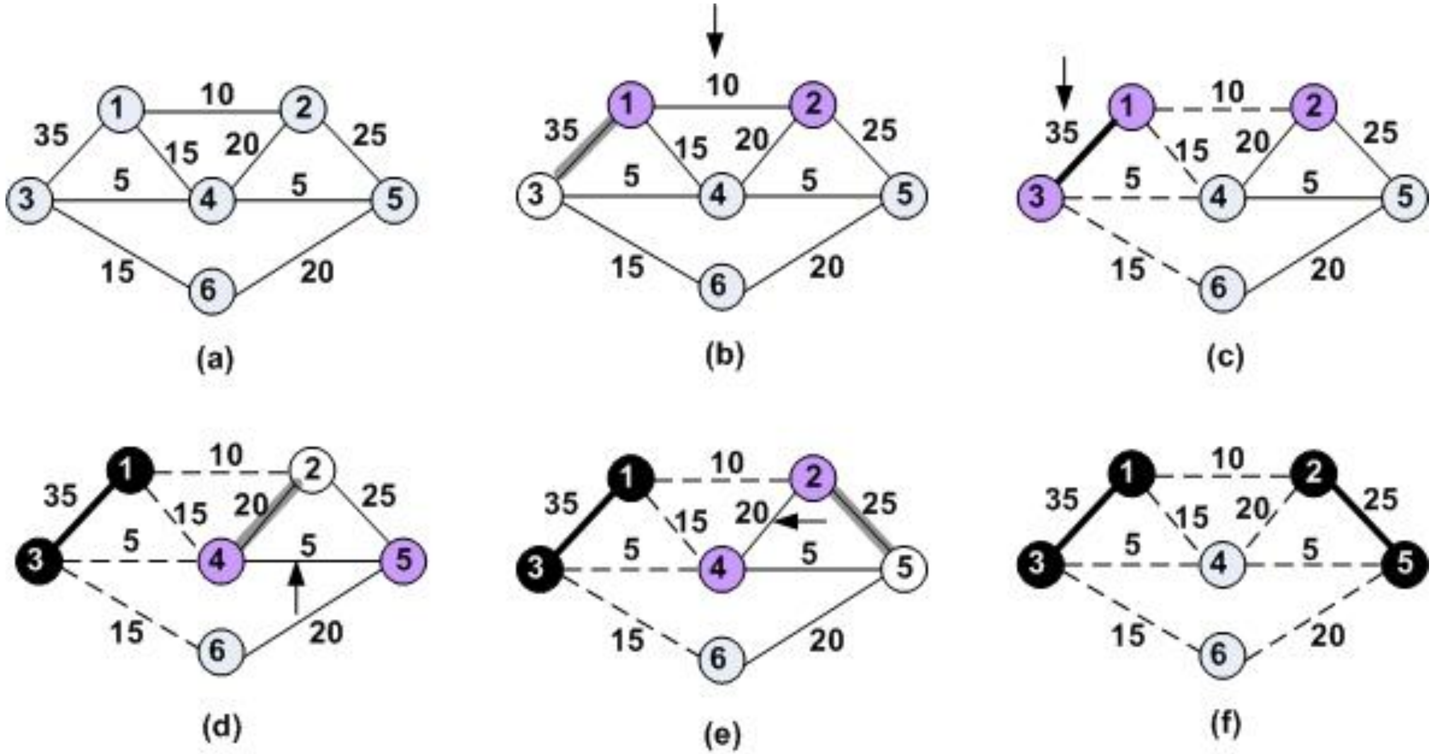
- Robert Preis's LAM algorithm: $O(|E|)$

Algorithm 7 Input: A graph G. **Output:** a matching M. **Effect:** computes a $\frac{1}{2}$-approx matching M in G.

```
procedure LAM(G = (V, E), w : E → R⁺, M)
    M ← φ;
    repeat
        Pick a locally-heaviest edge e_uv ∈ E;
        M ← M ∪ e_uv;
        Delete all edges incident on u and v from E;
    until E = φ;
end procedure
```

A Serial ½-approx Algorithm: Local

- Sample execution of LAM:



Sequential in nature.

Assumptions for Parallelization

- Vertex-oriented data structures for graph representation
- Graph distributed among processors via vertex partitioning
- Owner-computes Model: each processor owns a set of vertices that it is responsible for

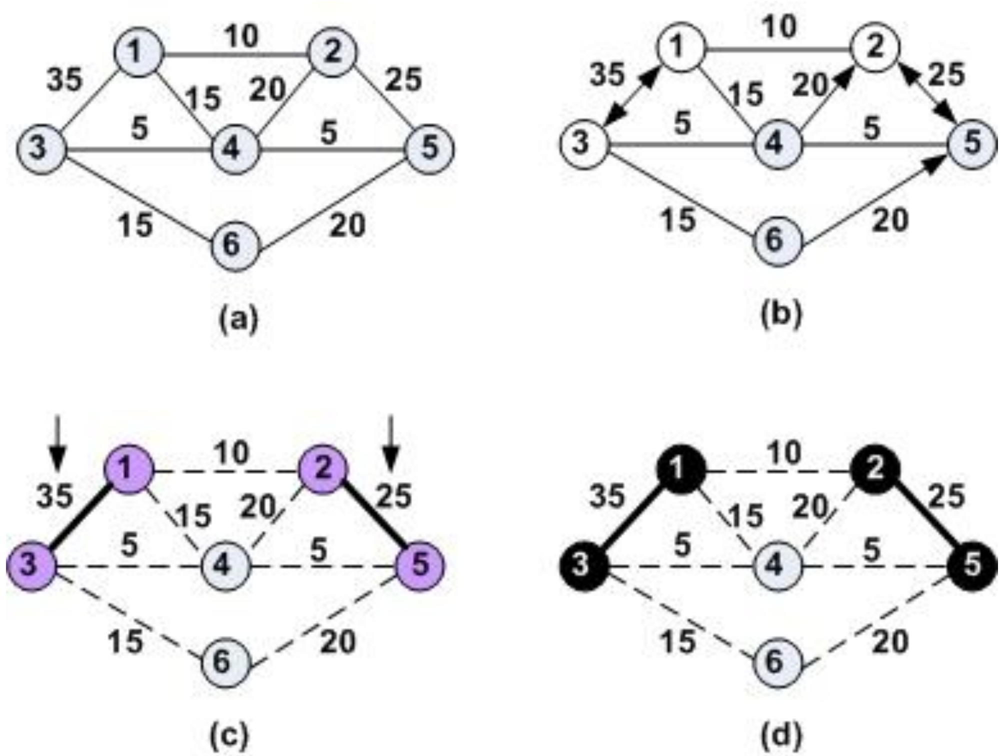
Towards Parallelization

Pointer-based algorithm:

1. For each vertex, set a pointer to the heaviest adjacent vertex.
2. If two vertices point to each other, then add these (locally dominating) edges to the matching.
3. Remove all edges incident on the matched edges, reset the pointers, and repeat.

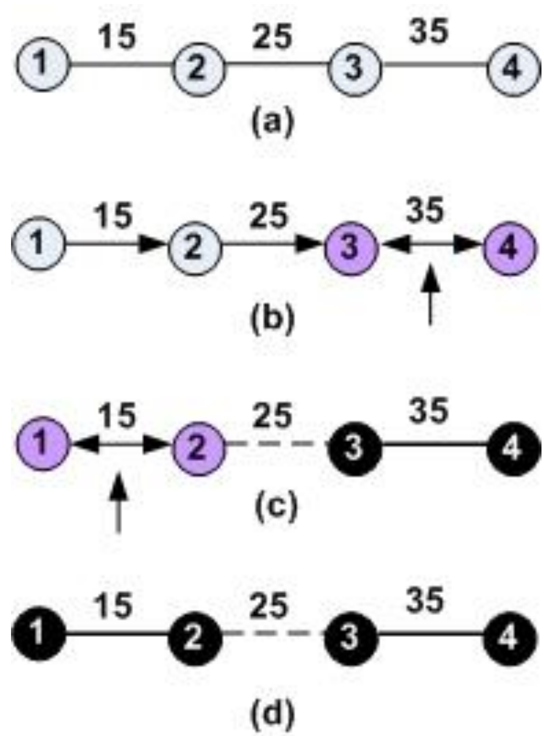
Towards Parallelization

- Sample execution of the pointer-based approach:



Parallel in nature.

A Worst-case Scenario



Forced sequentialness

Related Work (Pointer-based algorithm)

- 2004: Jaap-Henk Hoepman
  - Show parallel algorithm as a variant of Preis's algorithm
  - One vertex per processor (theoretical)
  - Algorithm converges in $\theta(2.|E|)$ messages
- 2007: Fredrik Manne and Rob Bisseling:
  - Extend Hoepman's work
  - Show parallel algorithm as a variant of Luby's algorithm
  - Complexity: $O(|V|d^2+|E|)$
  - No clear description of the parallel algorithm
  - BSP style

Note: Fredrik Manne independently developed the pointer-based algorithm that he presented at SIAM Parallel Processing 2006.

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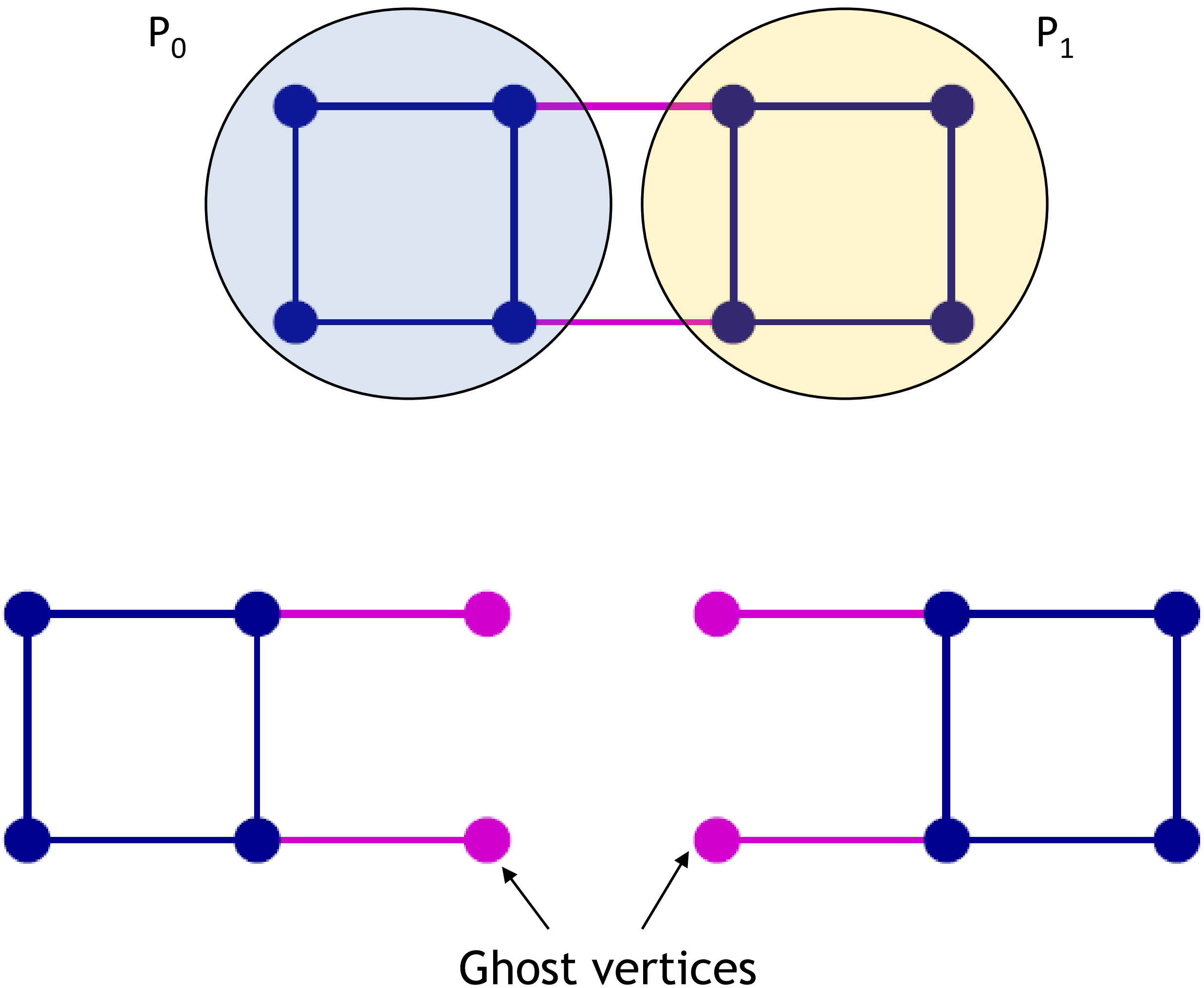
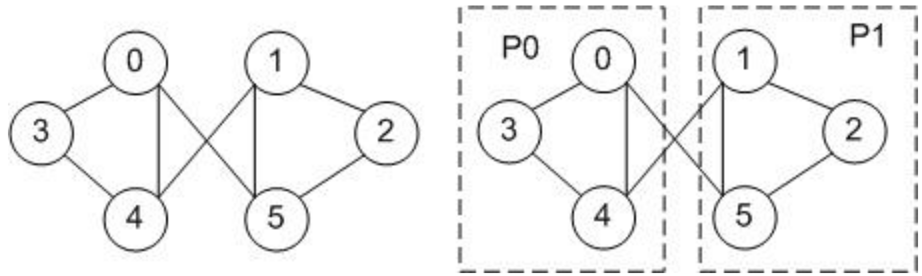
Data Distribution



Distributed Graph Data structure



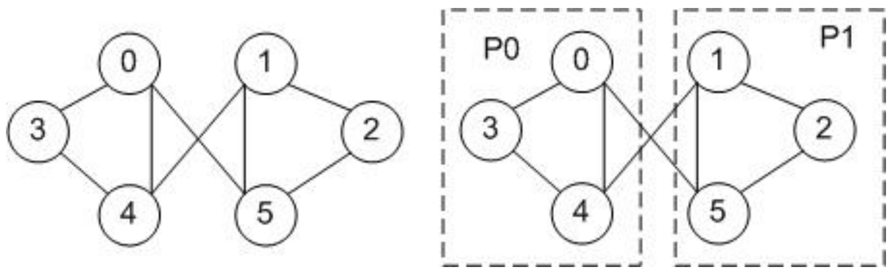
VtxPointer	0, 3, 6, 8, 10, 13, 16
Adjacency	(3, 4, 5)(2, 4, 5)(1, 5)(0, 4)(0, 1, 3)(0, 1, 2)
EdgeWt	w1, w2, w3, …

Compressed Storage Format

Vertex Indices	0	1	2	3	4	5
Owner Processor	P0	P1	P1	P0	P0	P1
New Indices	0	3	4	1	2	5

Vertex distribution and renumbering

Distributed Graph Data structure



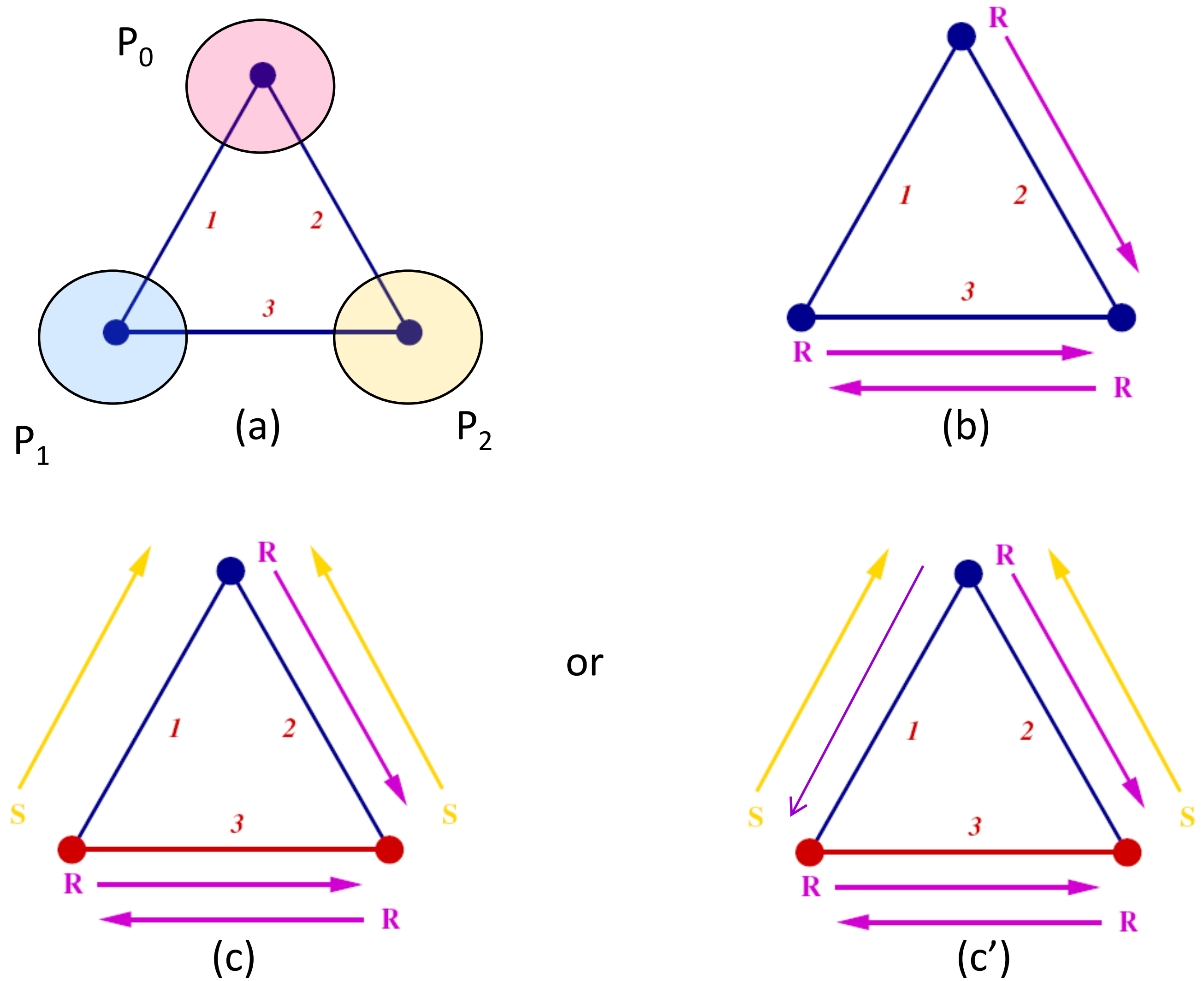
Processor 0:	Processor Pointer	0, 3, 6
	VtxPointer	0, 3, 5, 8
	Adjacency (m)	(1, 2, **5**)(0, 2)(0, 1, **3**)
	EdgeWt	e1, e2, e3, …
	VtxWt	v1, v2, v3, …
Processor 1:	Processor Pointer	0, 3, 6
	VtxPointer	0, 3, 5, 8
	Adjacency	(**2**, 4, 5)(3, 5)(**0**, 3, 4)
	EdgeWt	e1, e2, e3, …
	VtxWt	v1, v2, v3, …

Data structure on each processor

FindOwner(ghost-vtx): O(lg P); Storage: O(P)

A parallel algorithm:

Hoepman's algorithm with one vertex per processor



Our algorithm: many vertices per processor

1. Initialization: //(local computation)
   - Identify locally dominant edges
   - Send requests if needed
2. Computation: //(communication/computation)
   - Receive messages
   - Computation based on the received messages
   - Send messages is needed
   - Repeat until no more edges can be matched

Note: SPMD model; Distributed memory; Explicit messages

PART-1: Initialization

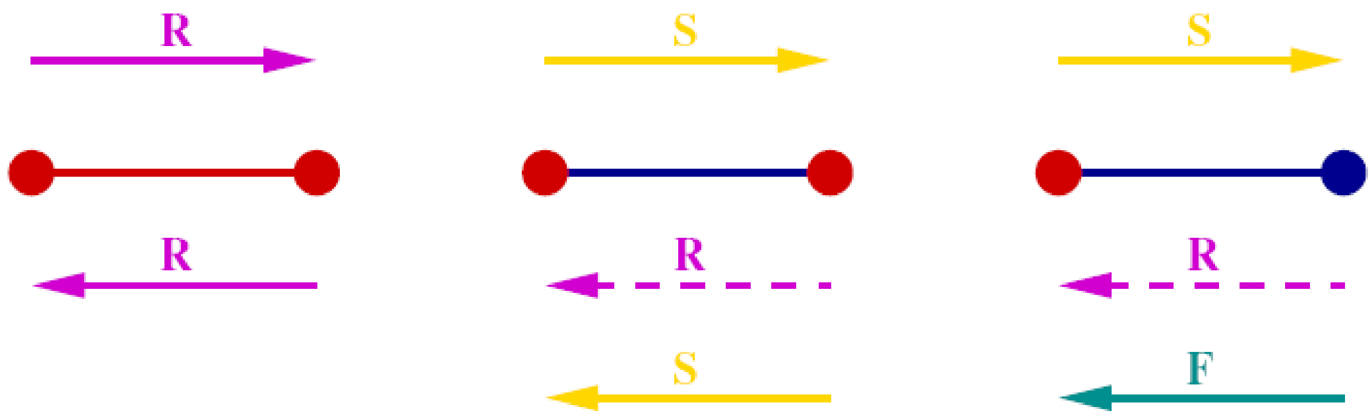
- For each vertex v_i set the pointer to the heaviest neighbor
  - If the heaviest neighbor is a ghost vertex, send a REQUEST message to its owner;   //Non-blocking
  - If v_i has at least one cross-edge incident on it:
    - S ← S U {v_i}
    - Counter[v_i] = #cross-edges incident on v_i
- Repeat:
  - For all vertex pairs that point to each other, add the corresponding edges to the matching
  - Remove edges incident on the matched edges (send SUCCESS messages)
  - Reset the pointers (send messages if needed)
  - Repeat until no more edges can be added to the matching

PART-2: Computation

- WHILE (S ≠ NULL) DO
  - Receive a Message //Blocking; from any source
  - Process the Message based on type
    - Request, Success, or Failure
    - Add to matching, and remove edges incident (send SUCCESS messages)
    - Reset pointers for vertices that were pointing to the matched vertices (Send messages if needed)
  - Update:
    - Counter[v_i]: Decrement the counter
    - S (remove v_i from S when Counter[v_i]=0)
    - Send FAILURE messages if some vertex cannot be matched

MPI standard requires that every SEND be matched with a RECEIVE. Therefore, we need set S and Counter[v] to keep track of all the messages that need to be received.

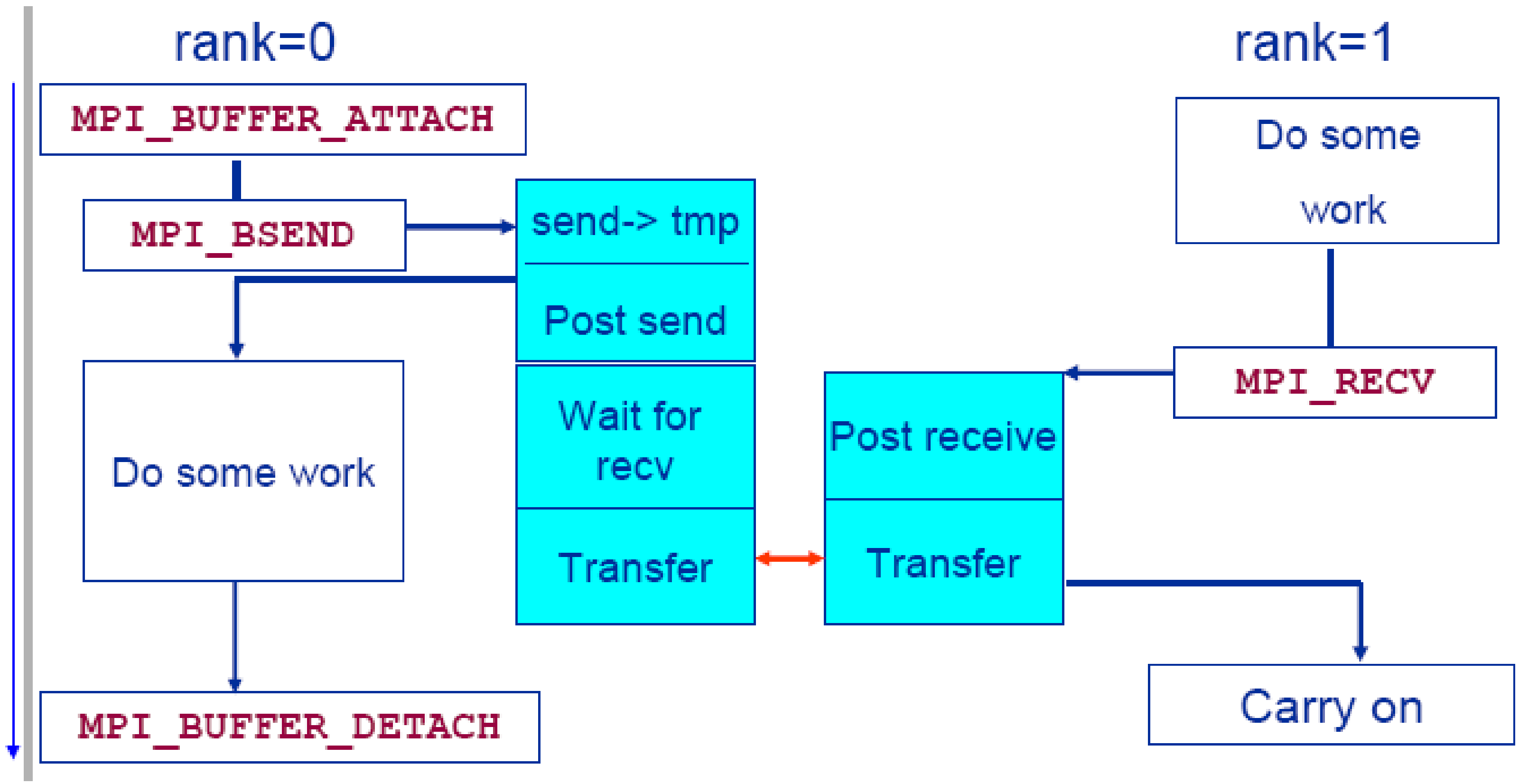
Communication Pattern



(a) (b) (c)

- Our scheme needs ≤ 3|EdgeCut| messages
- Can be optimized to 2|EdgeCut| messages

MPI: Buffered Sends



Source: Dr. Gerhard Wellein (RRZE) et al.

We also have an implementation with MPI_Isend() with similar performance.

Graph algorithms: Issues & Challenges

- Load balancing:
  - Pre-distributed data; 1D V/s 2D; performance of partitioners
- Locality:
  - Cache-aware V/s Cache-oblivious
- Ghost vertices:
  - Memory V/s Performance

"… computation done by **32,768** processors on BlueGene/L could be done by **five** to **10** processors of an MTA-2 with sufficient memory."

- Bruce Hendrickson and Jonathan Berry, "*Graph Analysis with High-Performance Computing*", Computing in Science and Engineering, IEEE and AIP, March/April 2008.

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Platform Details

- Zorka Compute Cluster:
  - *Compute Node*: Two dual core 3.0 GHz Intel Xeon (4 CPUs); 8 GB RAM
  - *Total Nodes*: 40 (160 cores)
- Network: Infiniband 4X (20 Gbits/s)
- Software:
  - Intel C++ compilers (–O2 –axT)
  - MVAPICH2, with 4 processes per node (wrap around if #processes > #cores)

We see about 20% performance difference between GigE and Infiniband.

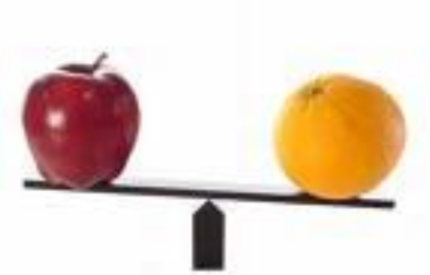
Test set 1: Matrices from Tim Davis Collection

Name	#Rows	#Cols	#nnz
Circuit Simulation			
Rajat16	94,294	94,294	476,766
Rajat21	411,676	411,676	1,876,011
Rajat29	643,994	643,994	3,760,246
Rajat30	643,994	643,994	6,175,244
Rajat31	4,690,002	4,690,002	20,316,253
ASIC_320ks	321,671	321,671	1,316,085
ASIC_680k	682,862	682,862	2,638,997
G3_circuit	1,585,478	1,585,478	7,660,826
Structural Engineering			
bodyy6	19,366	19,366	134,208
bcsstk38	8,032	8,032	355,460
bcsstk35	30,237	30,237	1,450,163
bcsstk39	46,772	46,772	2,060,662
crystk03	24,696	24,696	1,751,178
ct20stif	52,329	52,329	2,600,295
ptwk	217,918	217,918	11,524,432
Fluid Dynamics			
Pres_Poisson	14,822	14,822	715,804
af23560	23,560	23,560	460,598
Electrical Engineering			
onetone2	36,057	36,057	222,596
twotone	120,750	120,750	1,206,265
pre2	659,033	659,033	5,834,044

Name	#Rows	#Cols	#nnz
DNA Electrophoresis			
cage10	11,397	11,397	150,645
cage11	39,082	39,082	559,722
cage12	130,228	130,228	2,032,536
cage13	445,315	445,315	7,479,343
cage14	1,505,785	1,505,785	27,130,349
cage15	5,154,859	5,154,859	99,199,551
Chemical Engineering			
meg1	2,904	2,904	58,142
bayer04	20,545	20,545	85,537
bayer01	57,735	57,735	275,094
Economic Models			
g7jac040	11,790	11,790	107,383
g7jac080	23,670	23,670	259,648
g7jac160	47,430	47,430	564,952
g7jac200	59,310	59,310	717,620
Symmetric Indefinite			
F2	71,505	71,505	5,294,285
F1	343,791	343,791	26,837,113

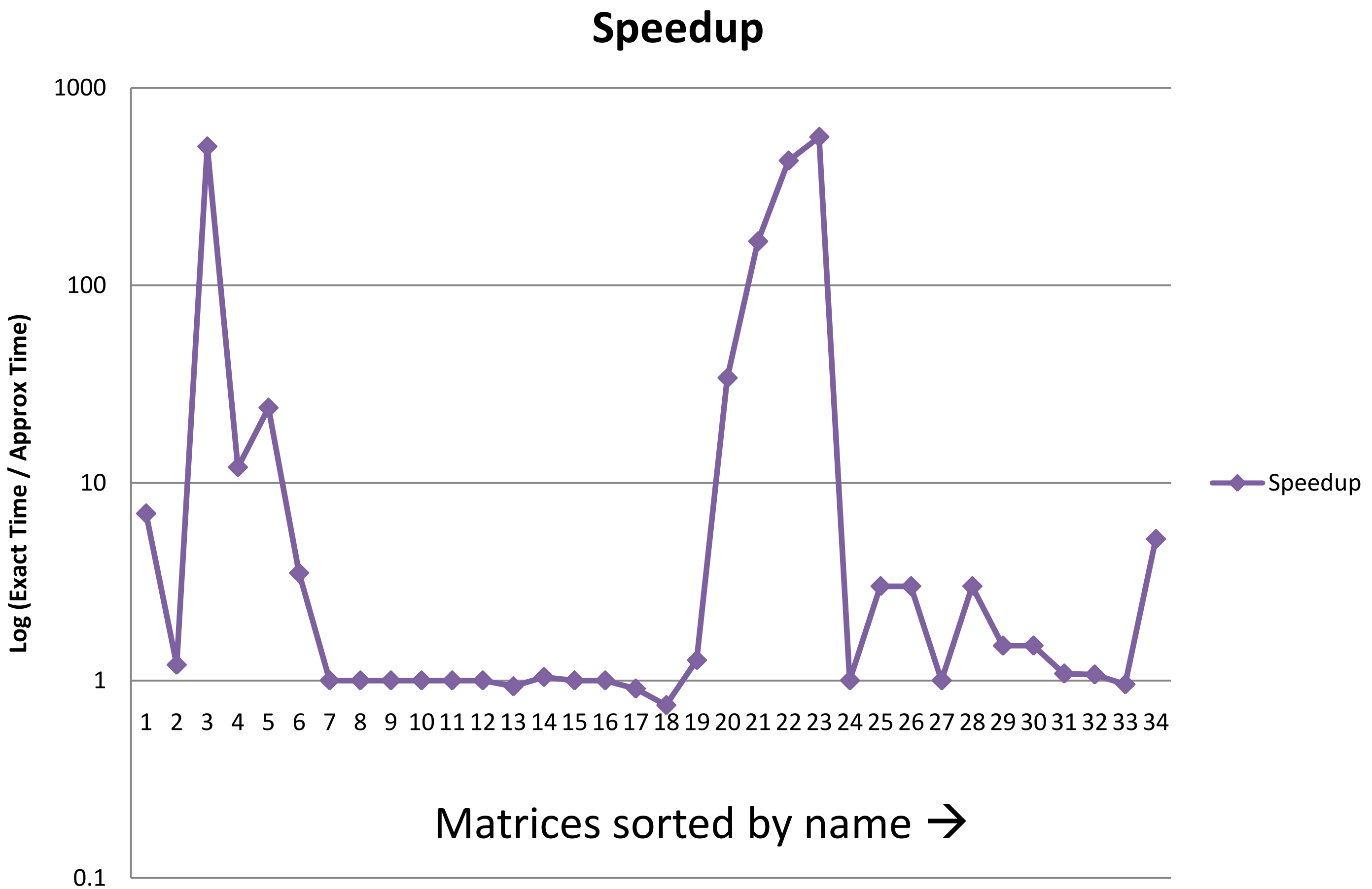
Performance of Sequential Algorithm

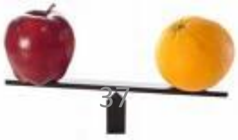
- Exact algorithm:
  - Perfect matching of maximum weight (similar to the algorithm implemented in MC64)
  - Binary heap data structure
  - Greedy initialization is critical for performance
  - $O(|V||E| + |V|^2\log|V|)$
- Approximation algorithm:
  - Pointer-based algorithm
  - $O(|V|d^2 + |E|)$
- Why?
  - Maximum weight matching is very slow
  - Context: Sparse matrix preconditioners



Comparison?

Performance: Execution Time





The approximation algorithm is very fast.

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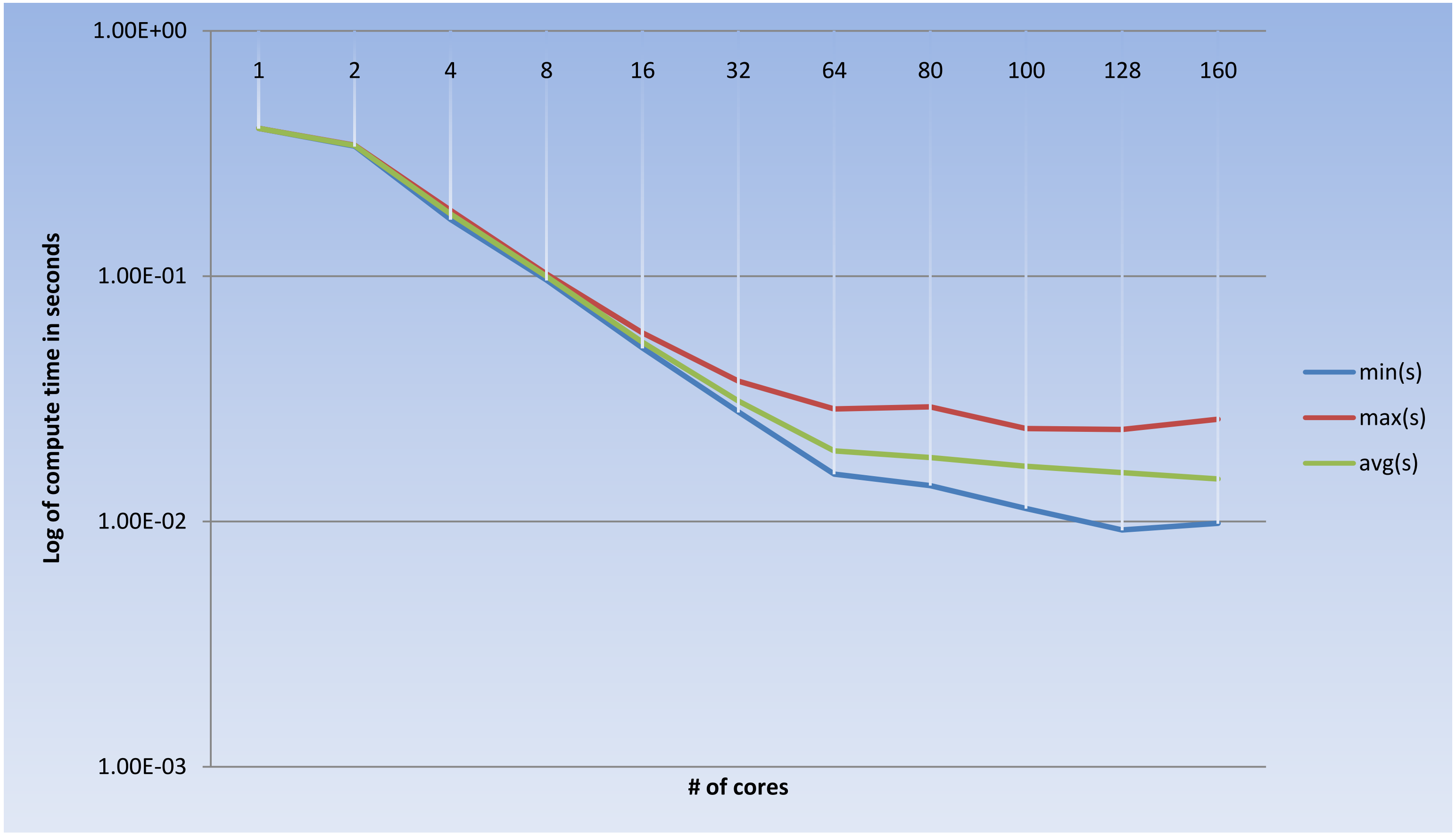
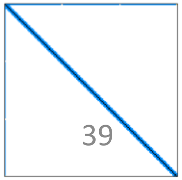
Rajat31: # NVtx: 9,380,004; #Edge: 20,316,253

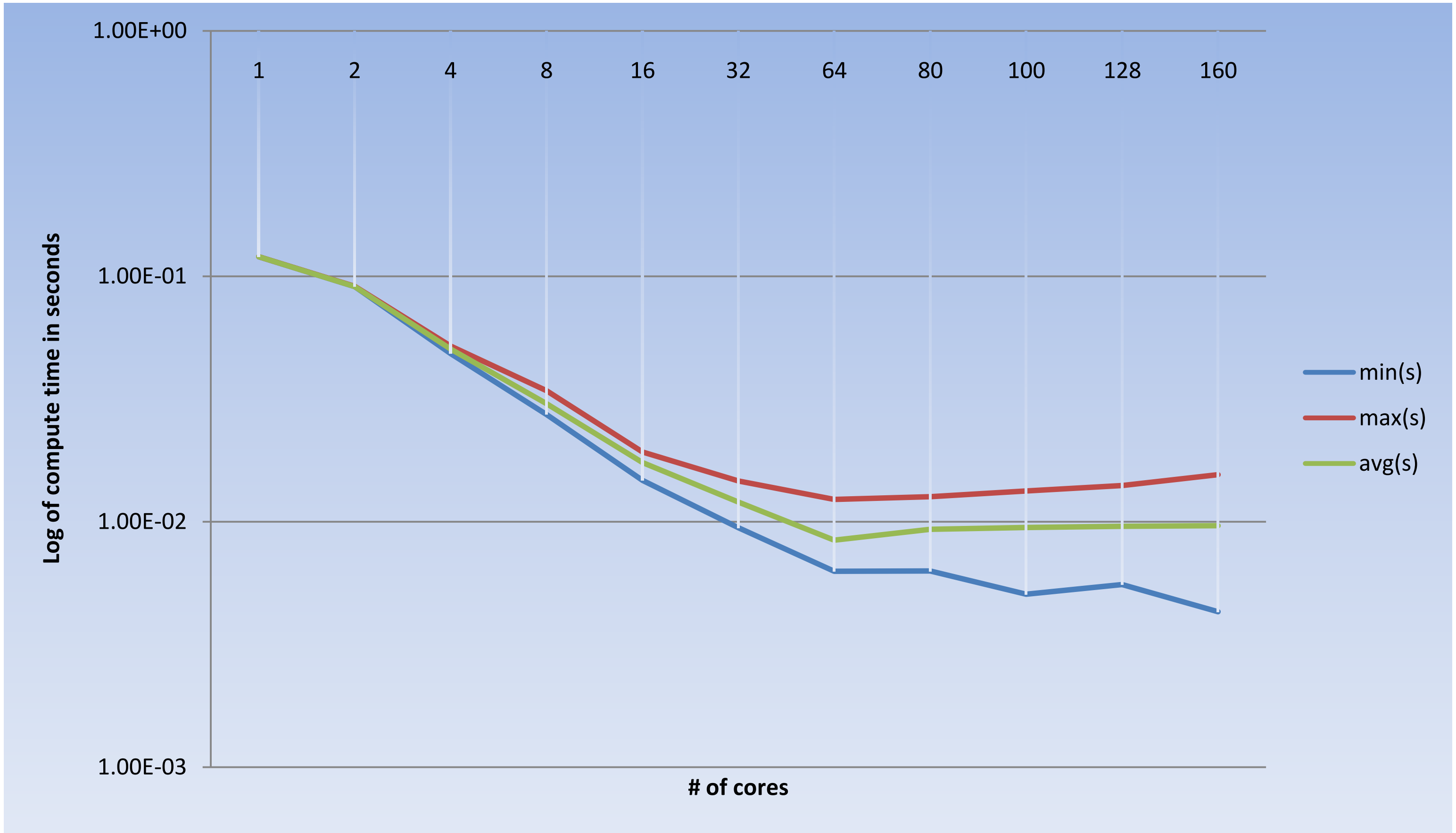


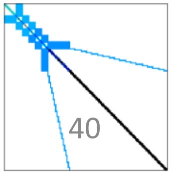
Min and Max times are the shortest and longest times on any given process (core). Avg is the average time of all the processes.

Matrix visualization from UF Sparse Matrix Collection

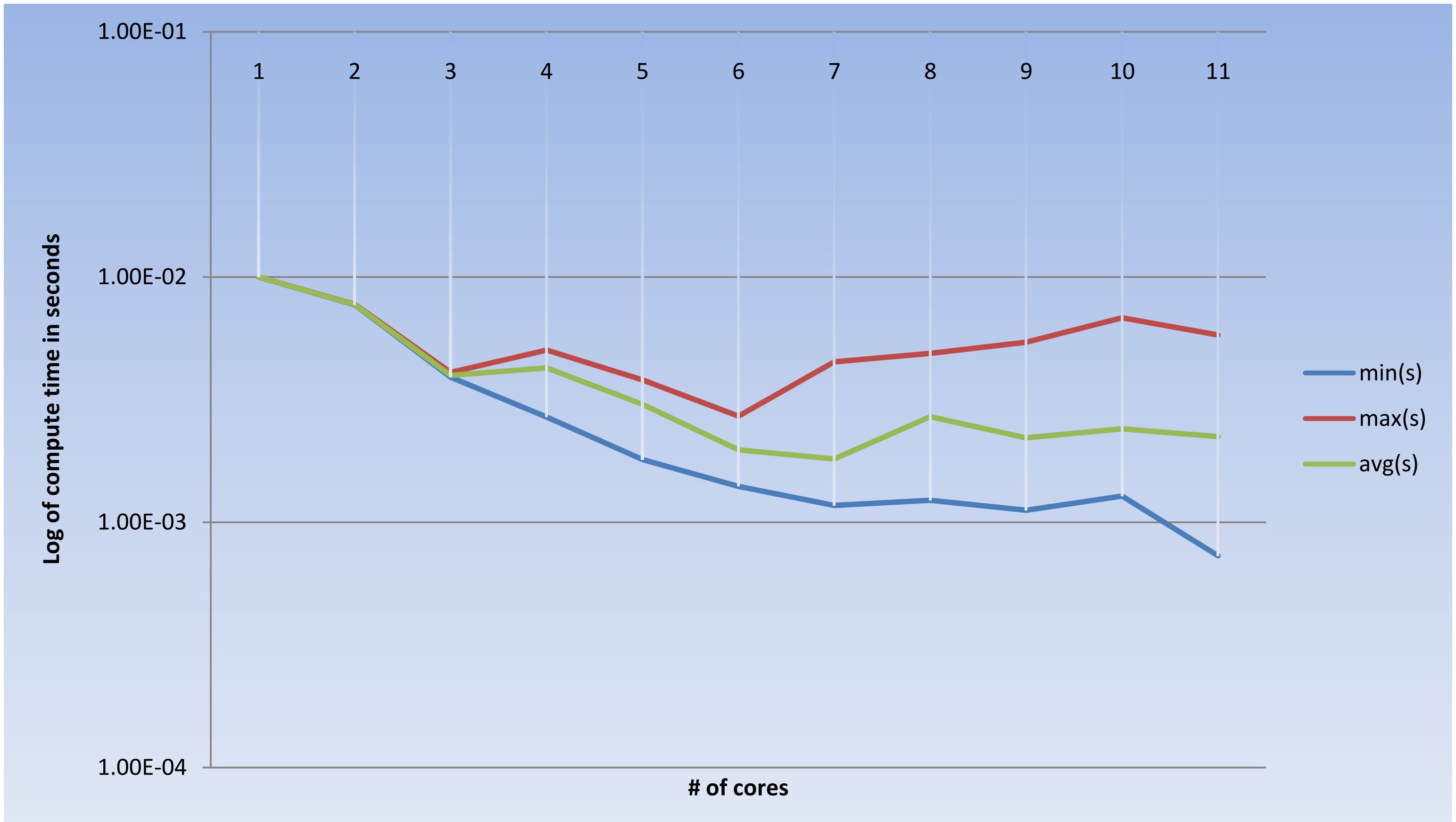


G3_Circuit: # NVtx: 3,170,956;   #Edge: 4,623,152

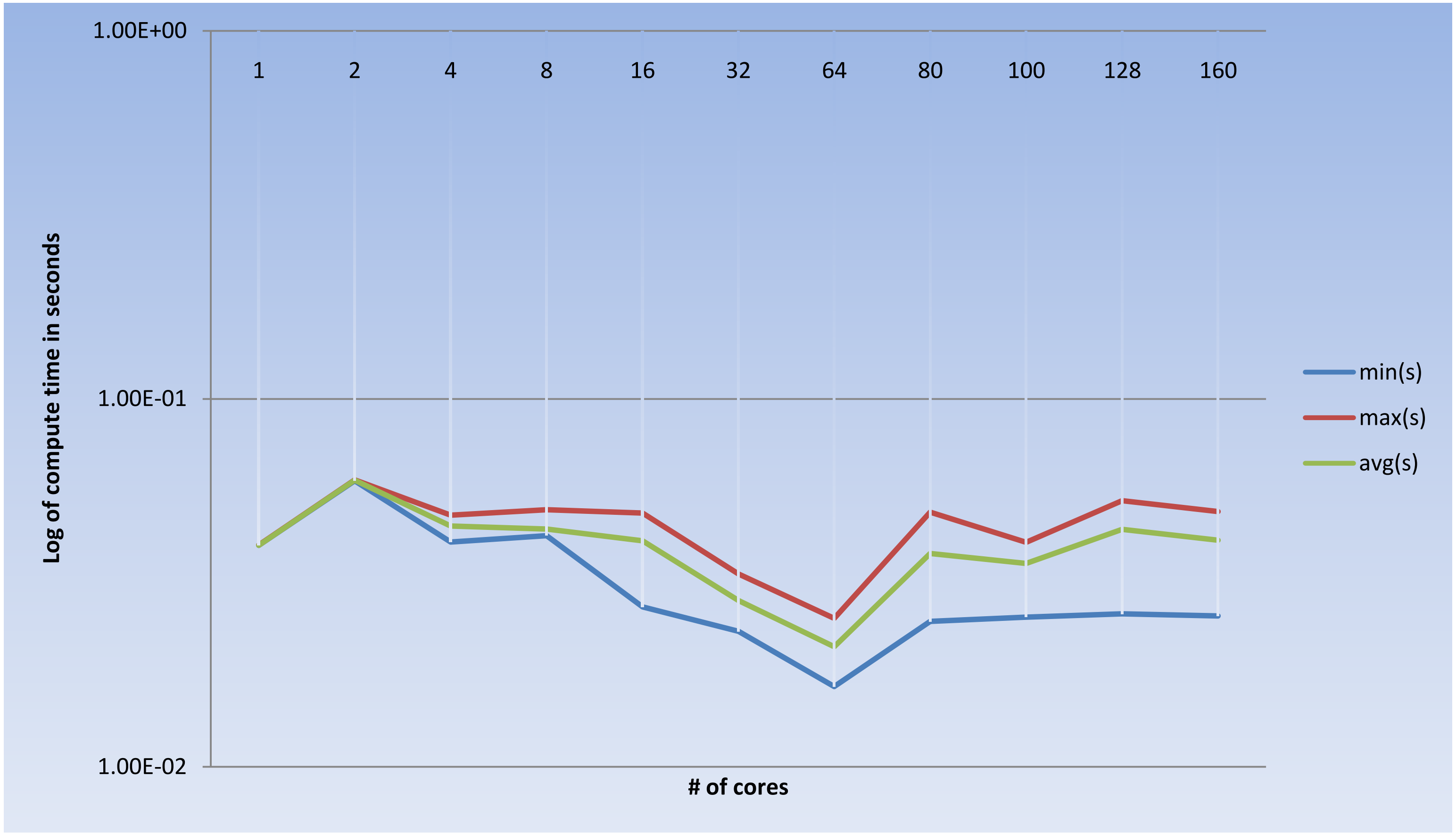


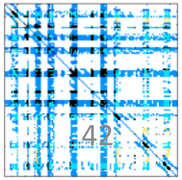


Bayer01: #Vtx=115,470; #Edges= 277,774

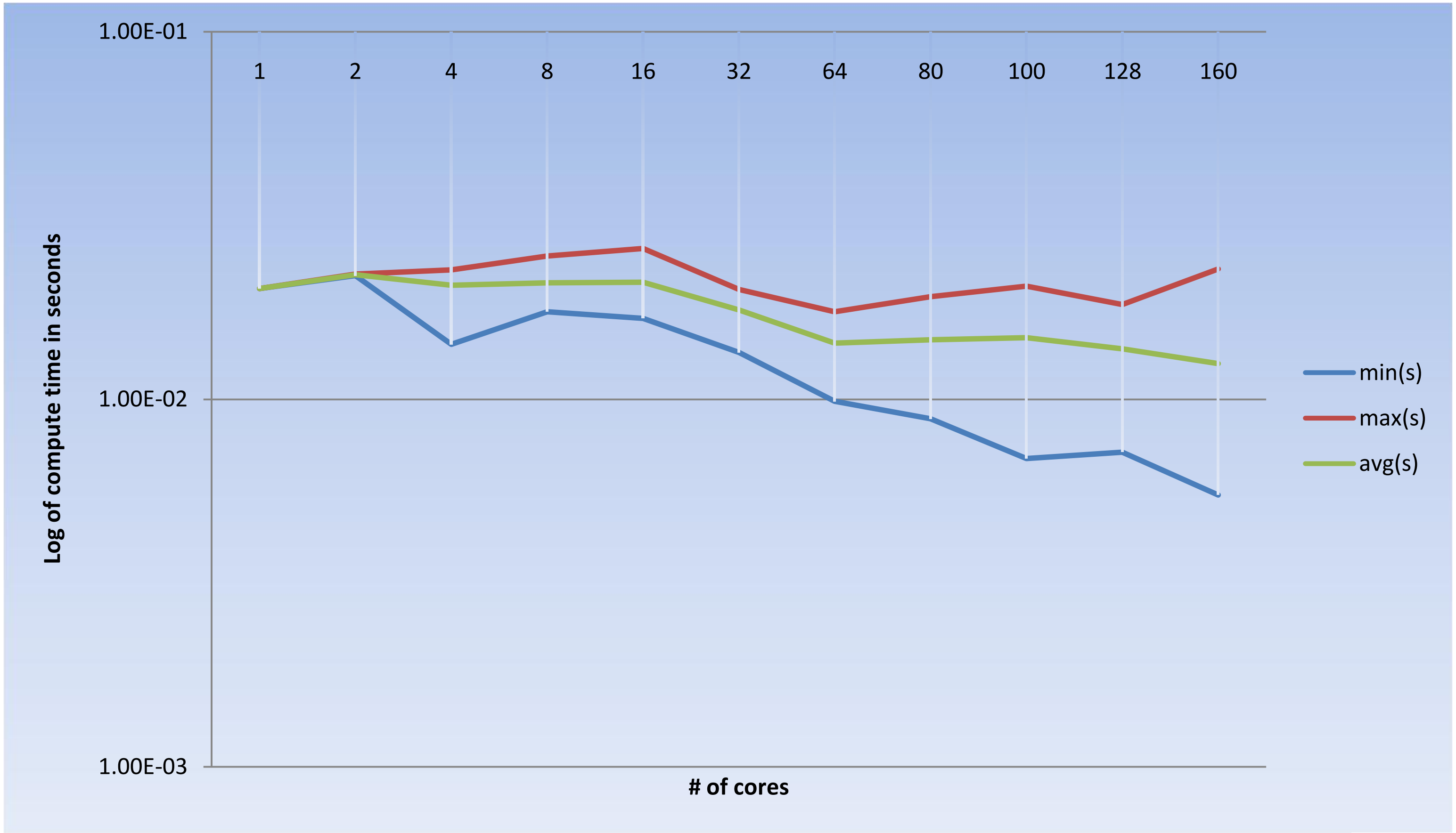


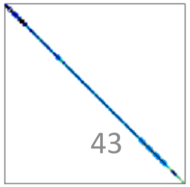
ASIC_320ks: #Vtx=643,342; #Edges= 1,827,807



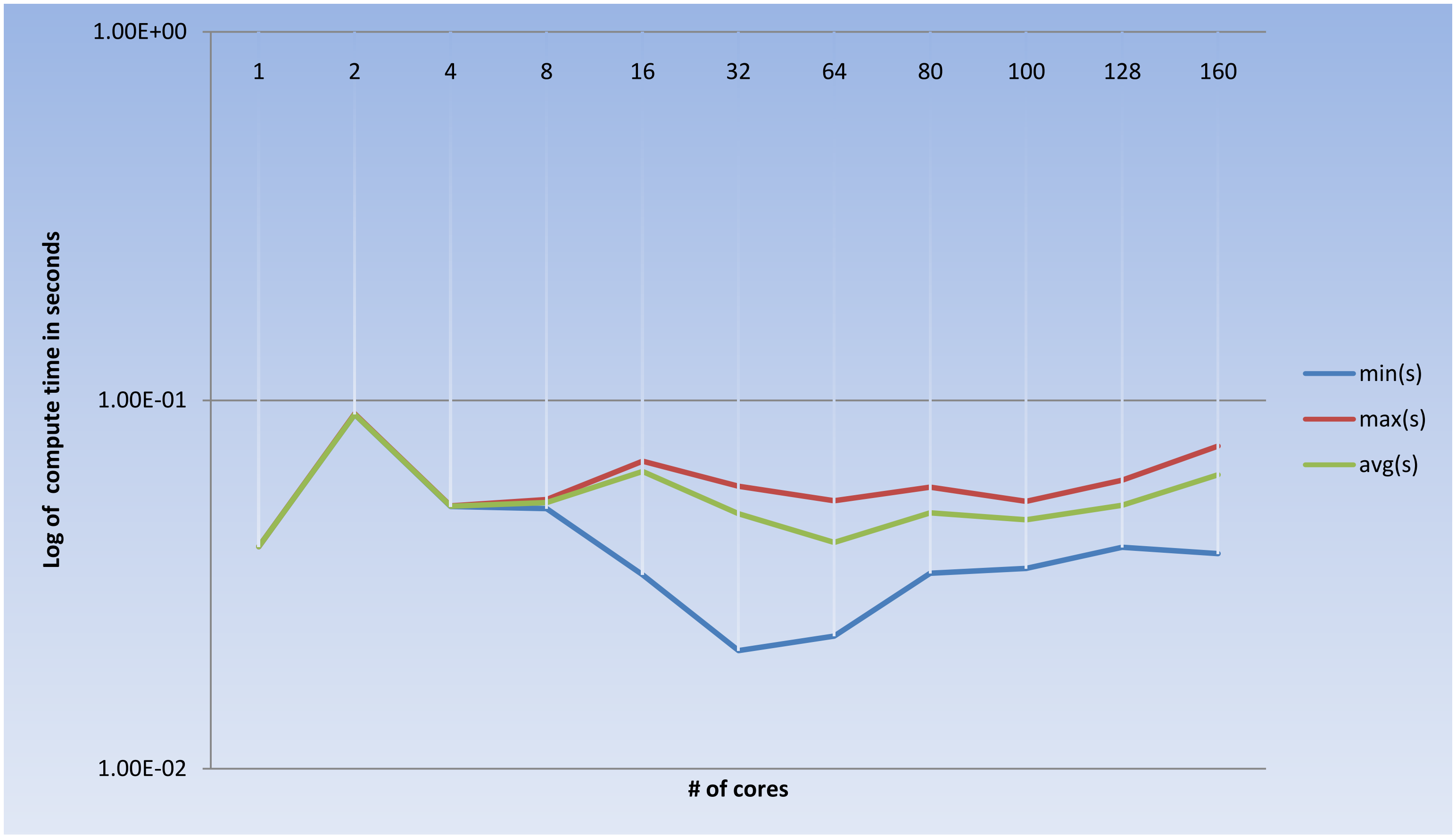


bcsstk39: #Vtx=38,732; #Edges= 77,057

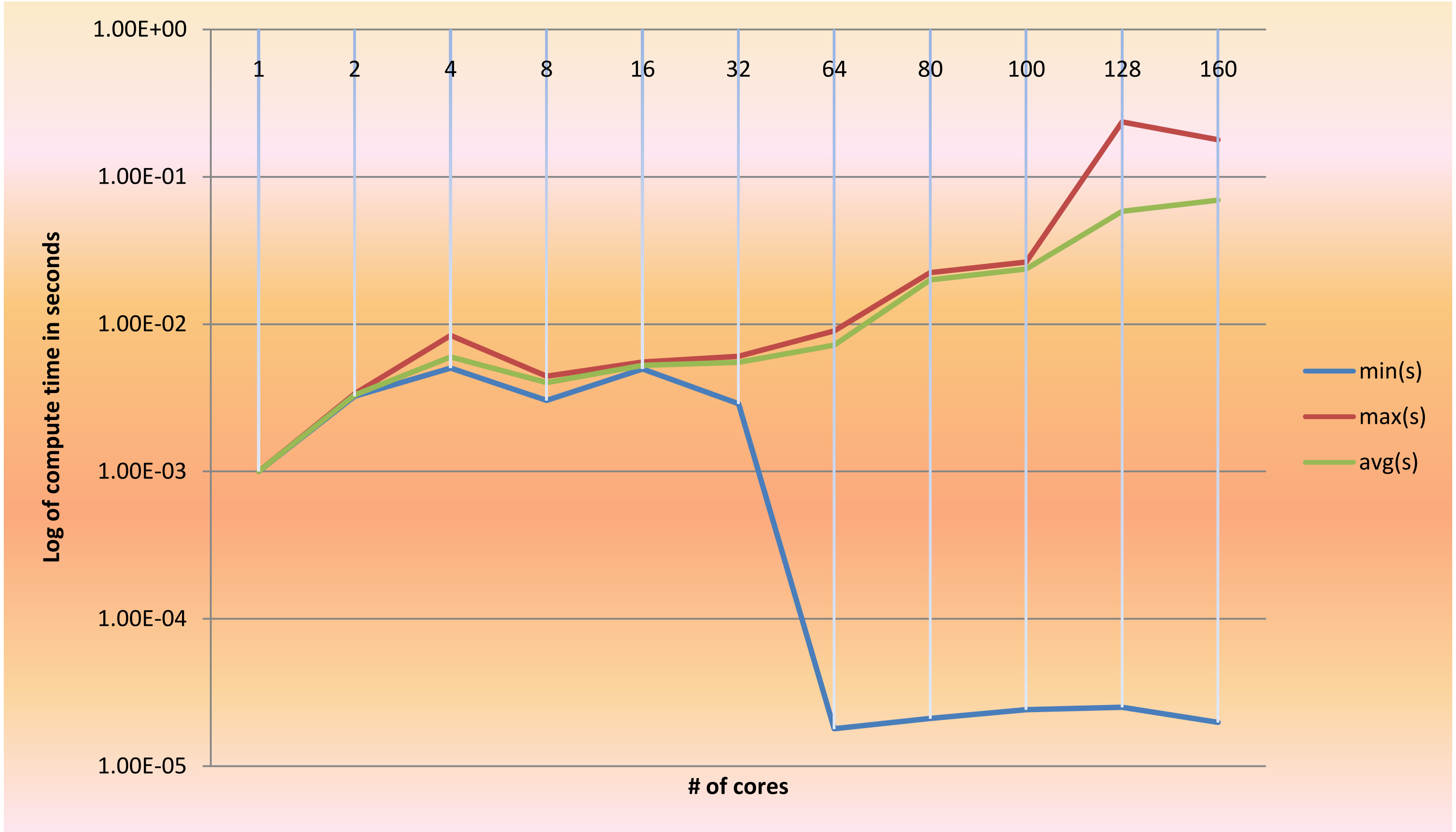


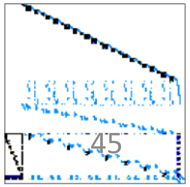


g7jac200: #Vtx=118,620; #Edges= 837,936

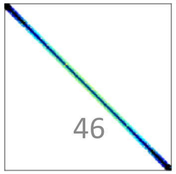


meg1: #Vtx=5,808; #Edges= 58,142

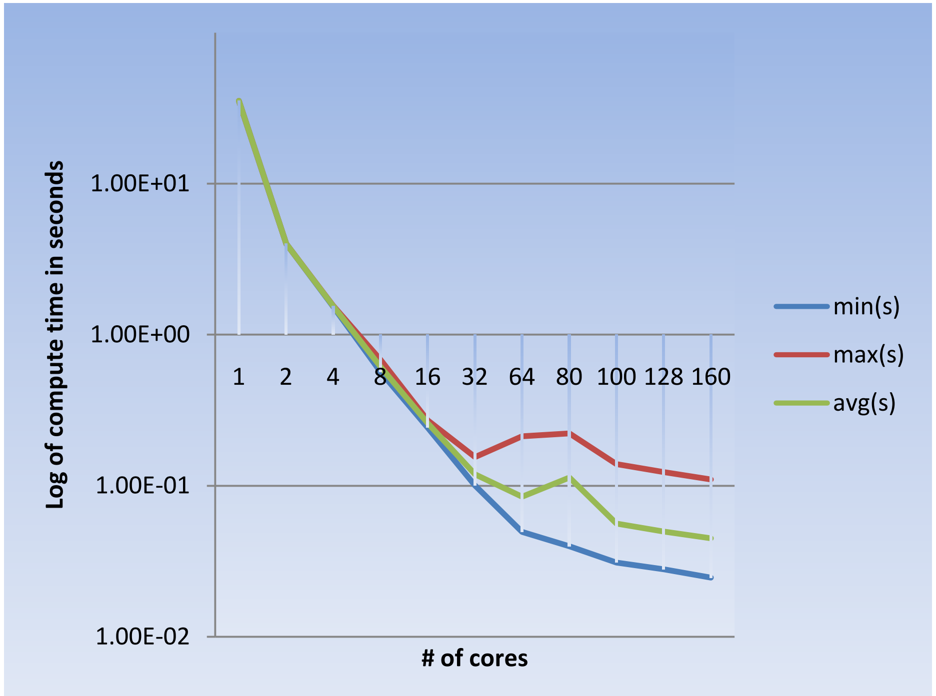


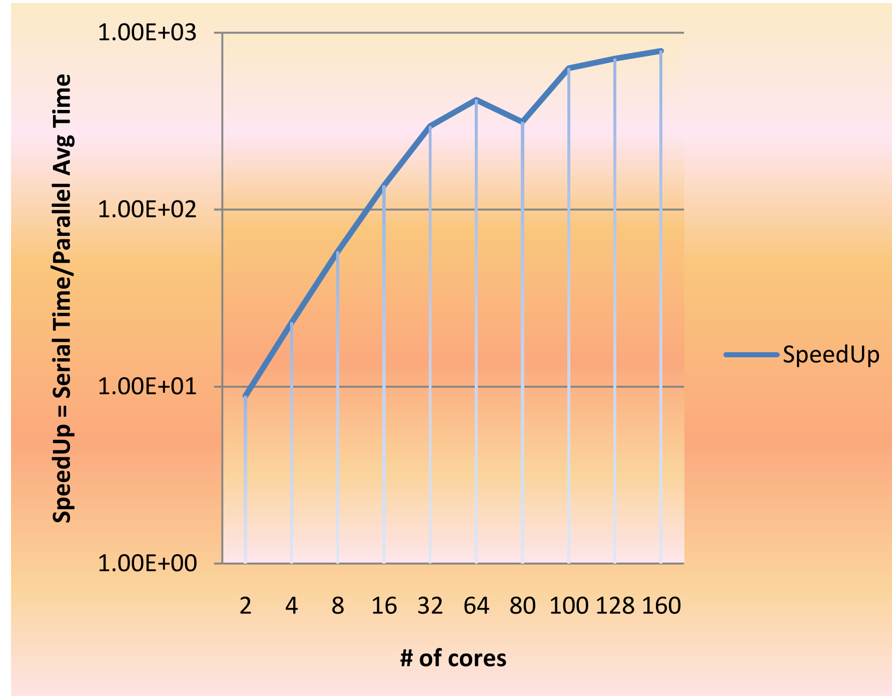


crystk03: #Vtx=49,392; #Edges= 887,937

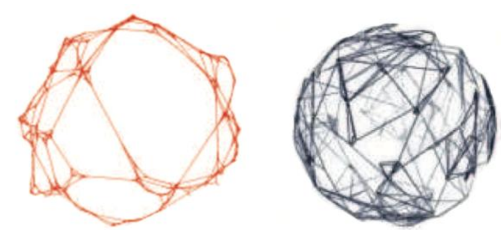


Synthetic Graph: SSCA#2





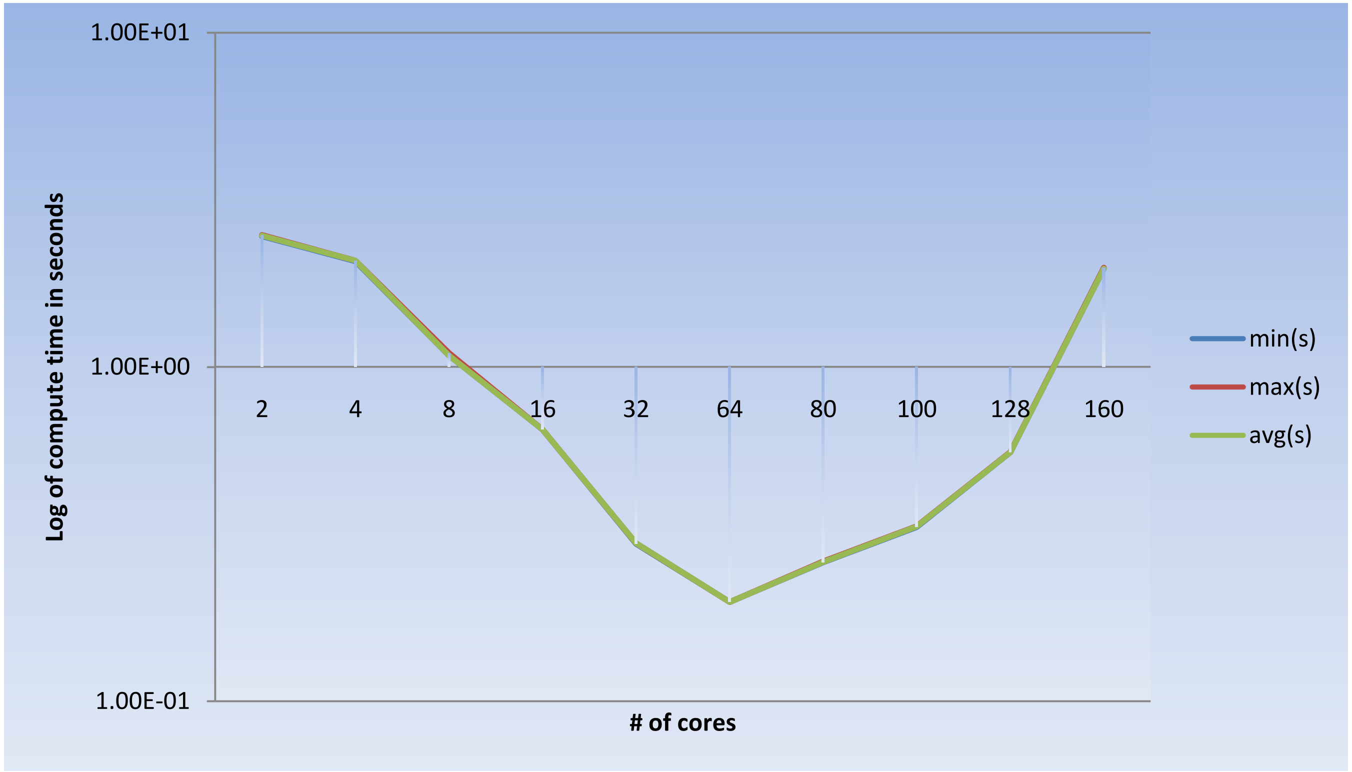
Graph: #Vtx: 2,097,152; #Edge: 63,148,387
Original graph generated with GT-Graph Generator.
Graph modified (treat it as bipartite graph) and duplicates eliminated.



Visualizing SSCA#2 graphs using Fiedler coordinates; Source: ctwatch.org

Super linear Speedup? Most probably due to cache effects than an inefficient serial implementation.

Synthetic Graph: Random Graph

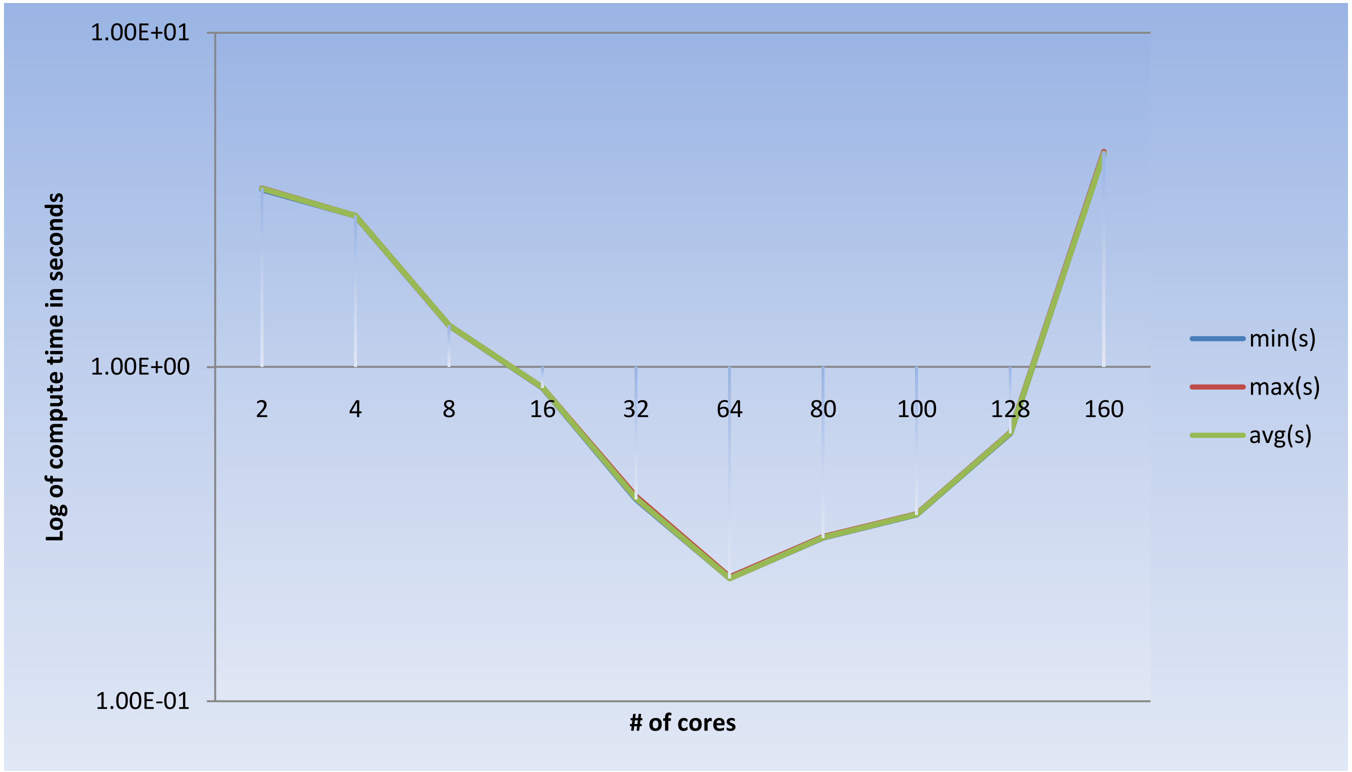


Graph: #Vtx: 500,000; #Edge: 1,500,000

Original graph generated with GT-Graph Generator.

Graph modified (treated as a bipartite graph) and duplicates eliminated.

Synthetic Graph: Random Graph



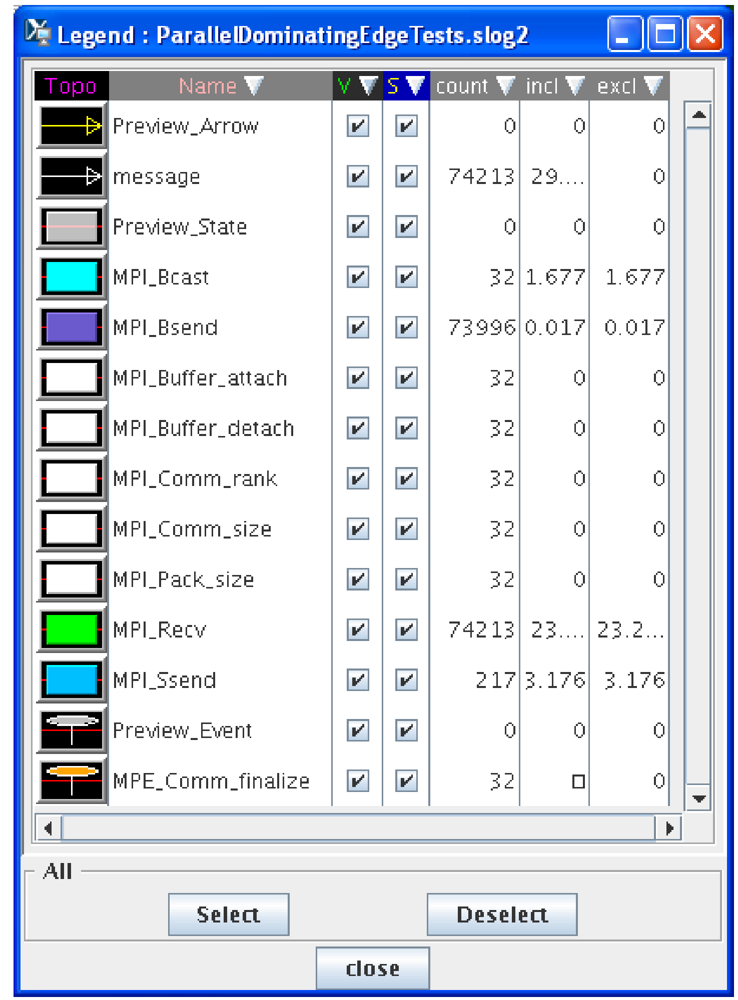
Graph: #Vtx: 1,000,000; #Edge: 2,250,000

Original graph generated with GT-Graph Generator.

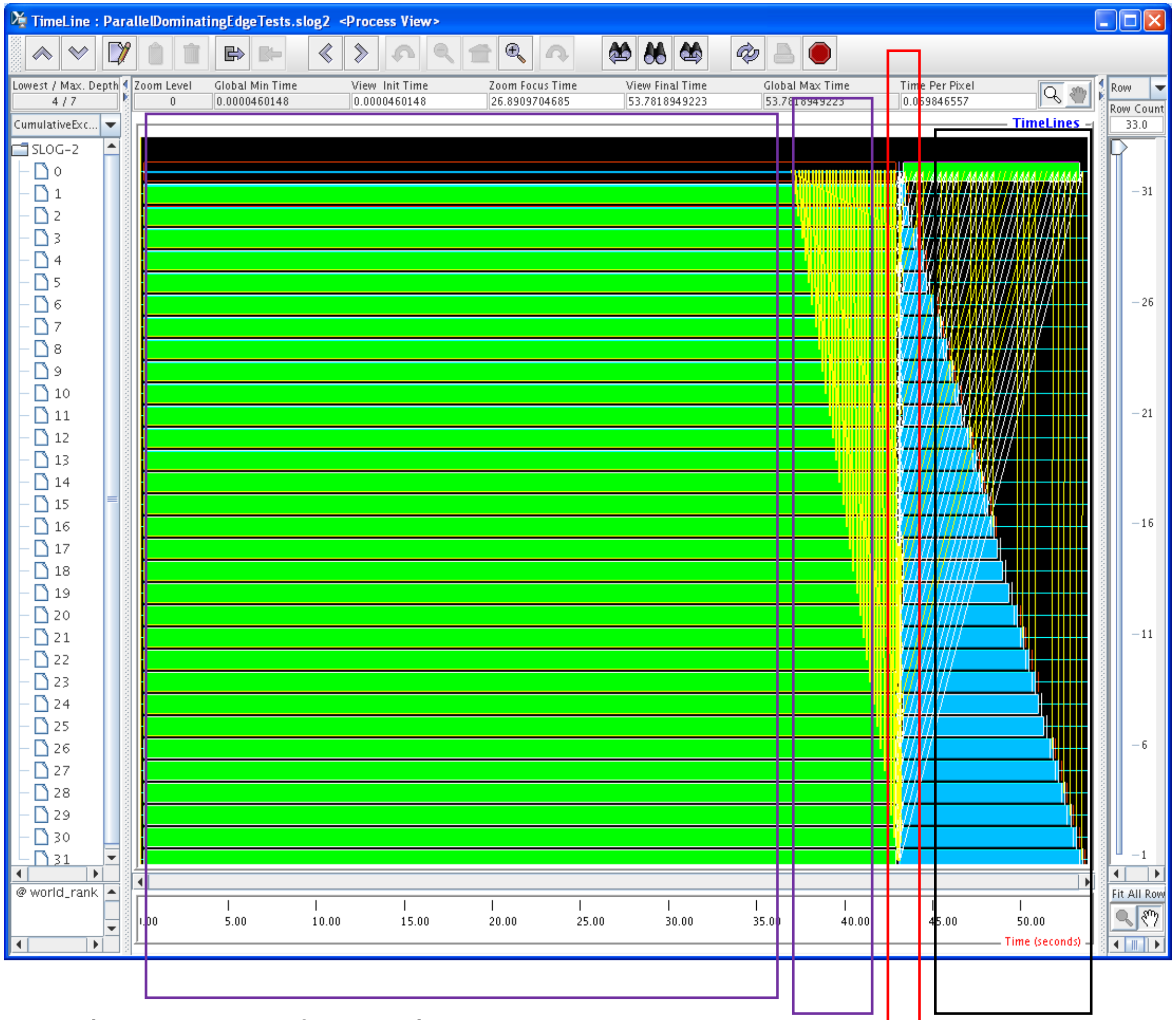
Graph modified (treated as a bipartite graph) and duplicates eliminated.

Jumpshot Pictures

- Input: Rajat31 (#Vtx: 9,380,004; #Edges: 20,316,253)
- Edgecut: 36,998; Transfer: 2.78 s; Weight: 6.25e+07; Cardinality: 4,688,751;
- Compute Time: Min: 2.79e-02; Max: 3.09e-02; Avg (32): 3.73e-02 seconds.



Entire Execution



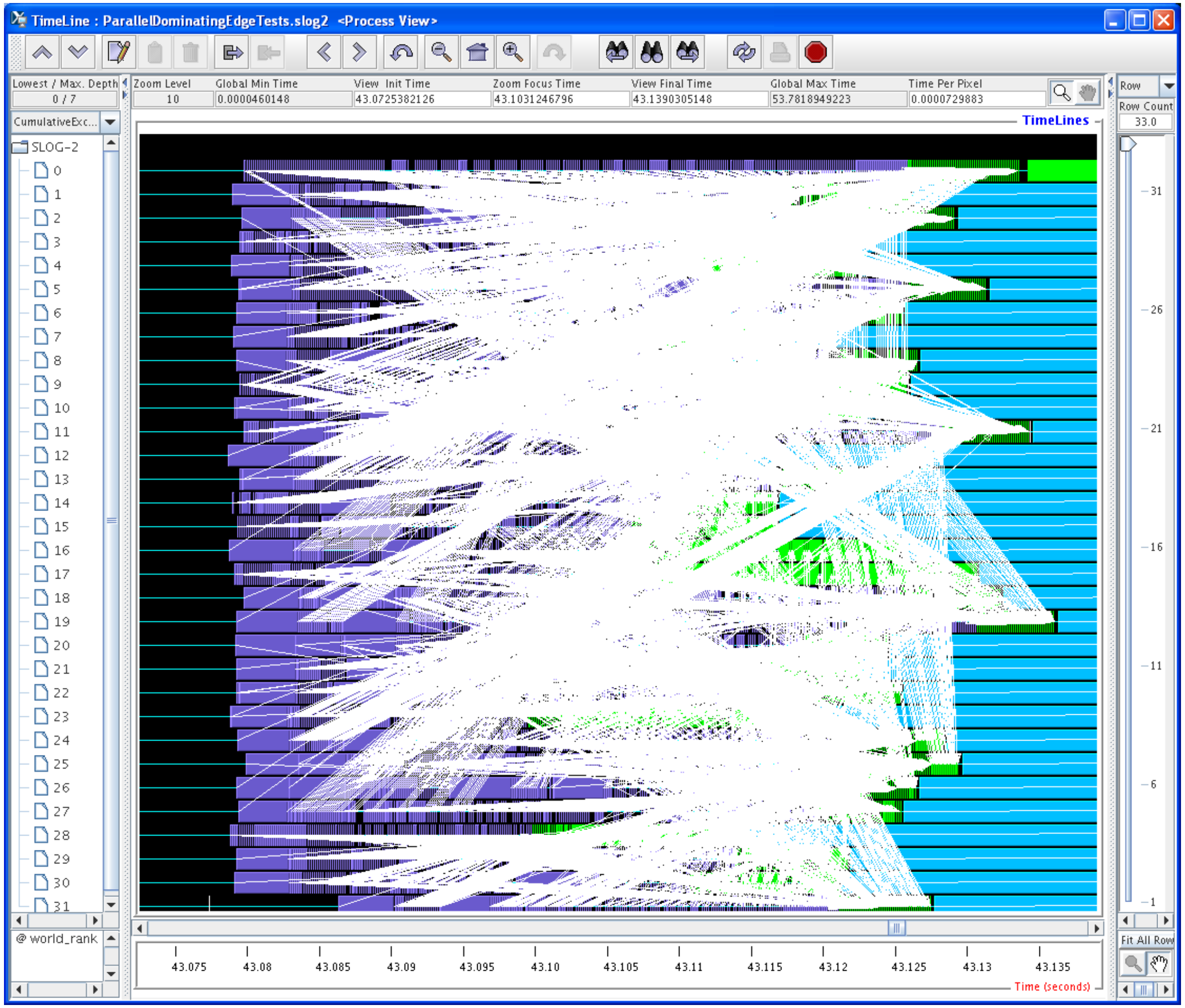
Initialization and Graph partitioning with Metis

Data transfer

Matching

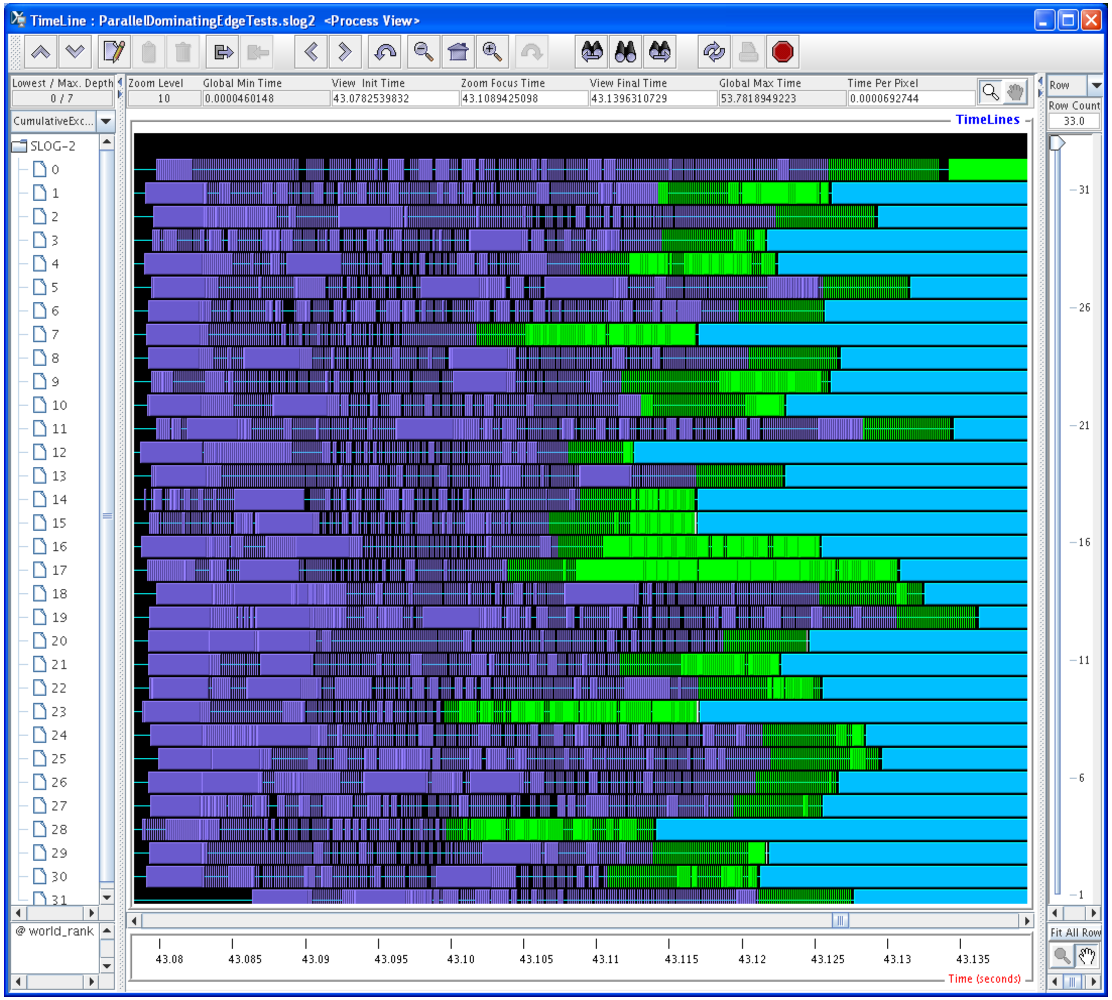
Communicate results

Close-up: Communication



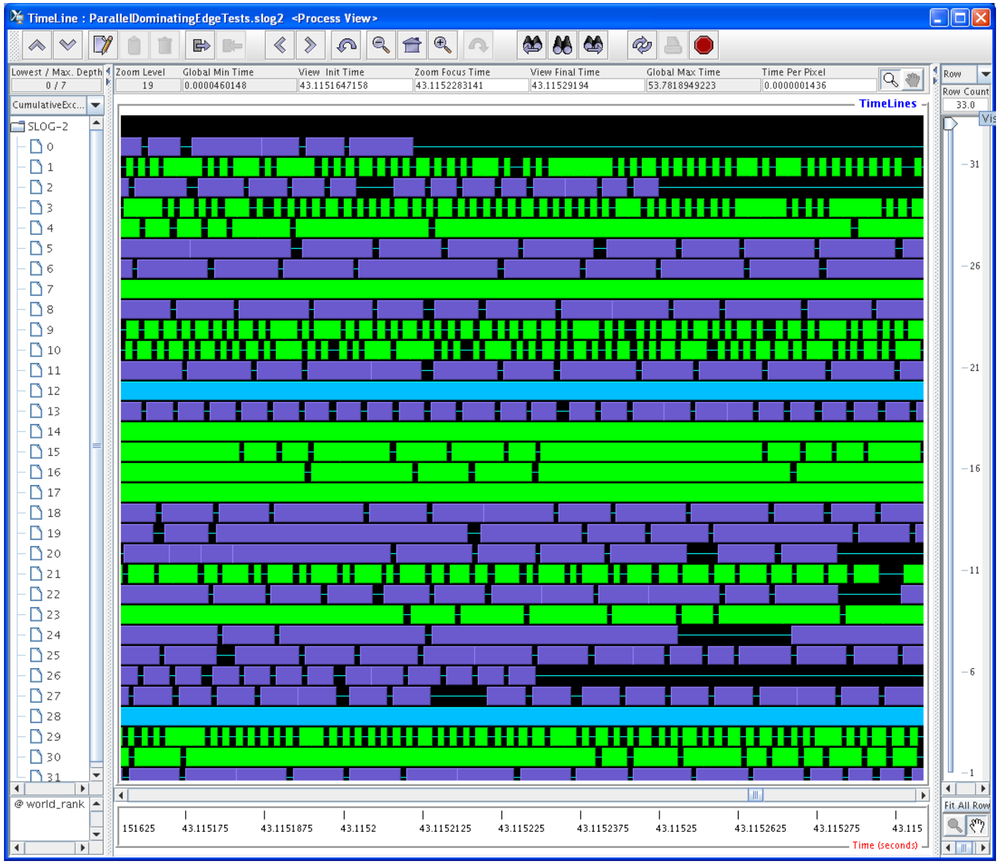
Legand: Purple = B_Send; Green = Receive

Closeup: Communication



Legand: Purple = B_Send; Green = Receive

Close-up: Communication



Long green bars: Waiting to receive → scope for improvement (speculation algorithms)

Outline

1. Introduction
2. Brief Survey of Parallel Matching Algorithms
3. A ½-approx Parallel Matching Algorithm
4. Computational Results
5. Conclusions and Future work

Contributions

- Extended the existing work
- Design of asynchronous communication scheme
- Efficient implementation for distributed memory system:
  - MatchBoxP
  - C++, STL, MPI

Conclusions

- Speedup is not a right goal for parallelization
- Graph structure and graph partitioning are critical for performance (but, probably, cannot be controlled)
- Memory limitations may change data structures, and therefore, performance
- One sided communications will probably help when used on systems with fast interconnects

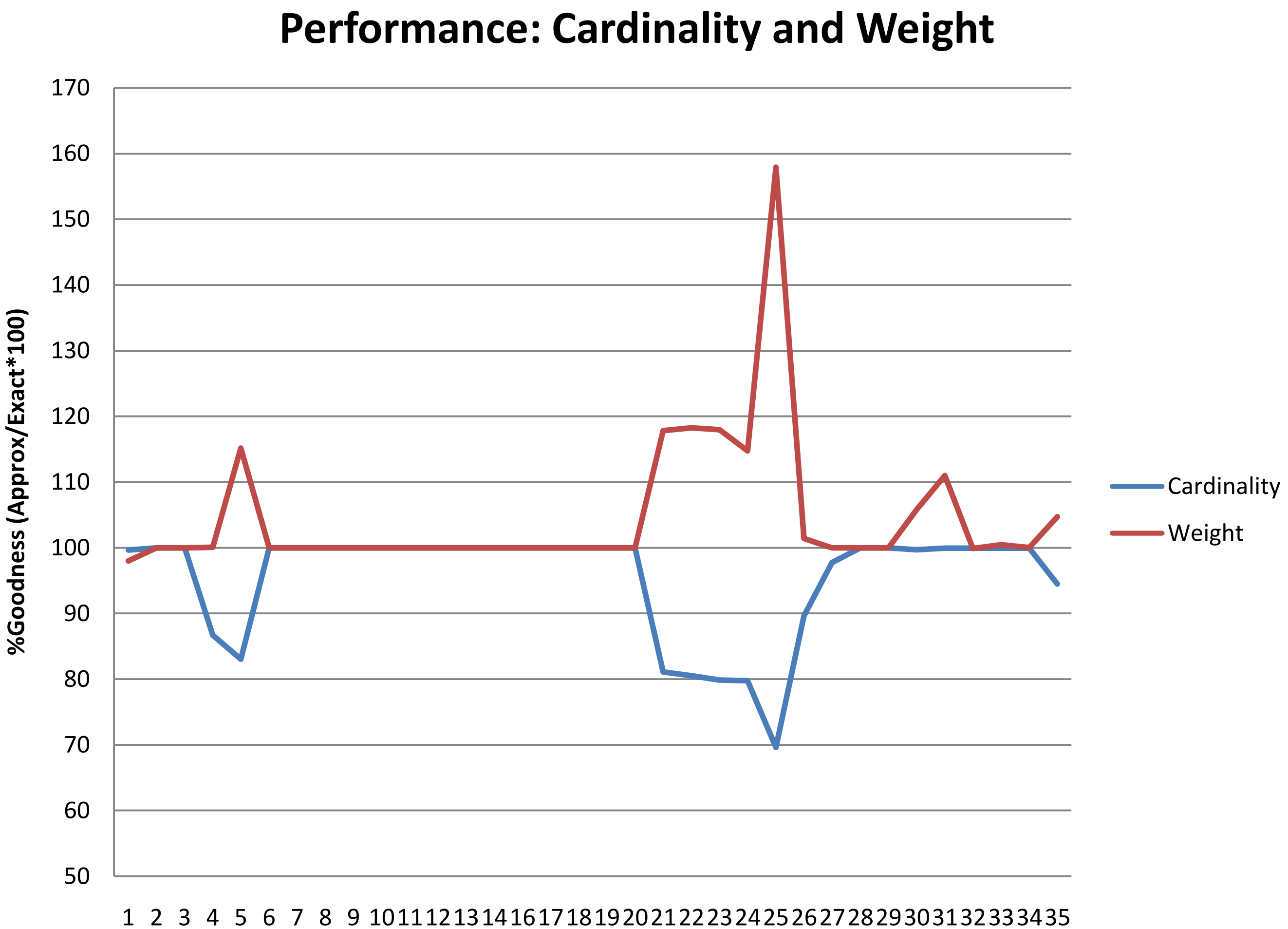
Future Work

- Tests for performance on the DOE Leadership-class machines (NERSC)
- Massive graphs
- Software engineering: data structures, error handling, documentation, etc.

THANK YOU !

We would like to thank Assefaw Gebremedhin for his time and suggestions to improve this work.

Performance: Cardinality & Weight



Matrices sorted by name →